## Liga: http://www.fraktalwelt.de/myhome/index.html

## Simple Iterations

Here we use an Iteration Machine with a single function. The calculation starts with a point $P(x ; y)$ of the two-dimensional plane. The next point is calculated with a simple formula in two variables $x$ and $y$. After that, we take the result $\mathrm{P}^{\prime}\left(\mathrm{x}^{\prime} ; \mathrm{y}^{\prime}\right)$ and use it for the following calculation. This iteration is repeated until the plotted points build the attractor of the process.


Here is the first example.

## Mira Attractor Applet

Use the Mira machine by changing the values in the edit fields at the bottom of the applet. Press the enter button and watch the new figure. The table below shows sample values. The figures depend sensitively on the numerals behind the decimal point. The color is changed every 1000 dots, until the 10 predefined colors are used, then the colors are repeated.

## The Formula

$$
\begin{aligned}
& \mathbf{x}^{\prime}=\mathbf{b} \cdot \mathbf{y}+\mathbf{f}(\mathbf{x}) \\
& \mathbf{y}^{\prime}=-\mathbf{x}+\mathbf{f}\left(\mathbf{x}^{\prime}\right) \\
& \text { mit } \\
& \mathbf{f}(\mathbf{x})=\mathbf{a} \cdot \mathbf{x}-(1-\mathbf{a}) \cdot \frac{2 \mathbf{x}^{2}}{1+\mathbf{x}^{2}}
\end{aligned}
$$

| a | b | Dots Scale | Name |
| :--- | :--- | :--- | :--- |
| 0.29 | 1.0005 | 50005 | Fort |
| -0.48 | 0.93 | 500010 | Wing |
| -0.4 | 0.9999 | 90007 | Amoeba |
| -0.2 | 1.000 | 80007 | Carpet |

Hints: Use $-1<\mathrm{A}<1$ and B near 1

Here we look at a second example. The third one is presented in the partition Iterations II.

## Kaneko Attractor Applet

Use the Kaneko machine by changing the values in the edit fields at the bottom of the applet. Press the enter button and watch the new figure. The table below shows sample values. If the figure is too big, decrease the scale factor. The radio checkboxes below choose the Kaneko square type I or the linear type II. The color is changed every 1000 dots, until the 10 predefined colors are used, then the colors are repeated.

## The Formula

$$
\begin{aligned}
\text { Typ I: } \mathbf{x}^{\prime} & =\mathbf{a} \cdot \mathbf{x}+(1-\mathbf{a}) \cdot\left(1-\mathbf{b} \cdot \mathbf{y}^{2}\right) \\
\mathrm{Typ} \amalg: \mathbf{x}^{\prime} & =\mathbf{a} \cdot \mathbf{x}+(1-\mathbf{a}) \cdot(1-\mathbf{b} \cdot|\mathbf{y}|) \\
\mathbf{y}^{\prime} & =\mathbf{x}
\end{aligned}
$$

A B Dots Scale Type
$\begin{array}{lllllllll}0.1 & 1.67 & 30000 & 160 & \text { I }\end{array}$
$\begin{array}{lllll}0.5 & 4 & 30000 & 180 & \text { I }\end{array}$
$\begin{array}{llllll}0.17 & 1.3 & 30000 & 200 & \text { II }\end{array}$
0.5622 .7630000250 II

Hint: Choose A, B > 0

## Iterations II - Hopalong

Here we have a look at the Martin-Attractors, also known as Hopalongs, a sort of orbitfractals. They are images of a simple two-dimensional iteration system. The name Hopalong is derived from the fact, that such an image is built of points hopping along on an elliptical path starting from one point in the center. Hopalong orbits were discovered by Barry Martin from the Aston University, Birmingham, England. A.K. Dewdney presented the Hopalongs in the magazine "Scientific American" (1986) and in Germany they became famous because they have been mentioned in the magazine "Spektrum der Wissenschaft".

| Imagen | a | b | c |
| :---: | :---: | :---: | :---: |
|  | 0.5 | -0.6 | 0.7 |
|  | -0.5 | 0.5 | 0.7 |
|  | 33 | 0.34 | 0.55 |
|  | -11 | 0.05 | -0.5 |
|  | 555 | 1111 | 555 |

(ref: http://www.fraktalwelt.de/myhome/simpiter2.htm\#)
The algorithm
An image is calculated using three parameters $a, b$ and $c$. Dewdney's article contains the following programme:

INPUT num
INPUT $\mathrm{a}, \mathrm{b}, \mathrm{c}$
$\mathrm{x}=0$
$\mathrm{y}=0$
PLOT(x, y)
FOR i = 1 TO num
$x x=y-\operatorname{SIGN}(x) *[\operatorname{ABS}(b * x-c)]^{\wedge} 0.5$
$y y=a-x$
$\mathrm{x}=\mathrm{xx}$
$y=y y$
ABS is the absolute value function. $\operatorname{SIGN}(\mathrm{x})$ is the same as $\mathrm{x} / \mathrm{ABS}(\mathrm{x})$. If $\mathrm{x}>0$ then $\operatorname{SIGN(x)}$ $=1$, if $x<0$ then $\operatorname{SIGN}(x)=-1$ and if $x=0$ then the result of $\operatorname{SIGN}(x)$ is zero, too.

After a certain number of points the color is changed. Normally The image doesn't depend on the first point. These attractors have the butterfly effect. That means if you change the parameters a bit, you'll get a total new image with very little similarity to the first one.


Screenshot of the Hopalong Program

## Iterations III - Hopalong and Mira in variations

If you define x and y as screen coordinates and $\mathrm{a}, \mathrm{b}$ and c as fixed values, then the iteration of Barry Martin's formula
$\mathrm{xn}=\mathrm{yn}-1-\operatorname{SQRT}(\operatorname{ABS}(\mathrm{b} * \mathrm{xn}-1-\mathrm{c})) * \operatorname{SIGN}(\mathrm{xn}-1)$
$\mathrm{yn}=\mathrm{a}-\mathrm{xn}-1$
leads to the well known Hopalong patterns. This bahavior changes dramatically, if you put these formulas into the drawing algorithm of a Mandelbrot program.

- Gallery: Hopalong- and Mira Variations


The varied algorithm
If the parameters $a$ and $b$ are not seen as constants, but now - in a nested loop - used as screen coordinates, combined with the variable x as a color value depending on the number of iterations, you'll see new interesting patterns. These patterns seem to be a weird mixture of interfering ribbons and frost tracery. They don't look like typical fractal structures with its self-similarity. Every part of the drawing plane has its own pattern structures, which don't seem to be related with one another.

Detailed articles


Hopalong varied
What is discribed above in a short manner, you can read much more detailed in a long article. But, sorry, you have to read it in German.

1. Hopalong and the Mandelbrot set

This article describes the origin of the images in the gallery and its imaginative interpretations.
2. Program descriptions

This article contains not only operating instructions, but also a comprehensive view of the algorithms.

Doenload programs
Three programs with the described variations are provided by Kurt Diedrich:

1. Hopalong-Classic
2. Hopalong-Special
3. Mira-Special

## Credits

The contents of this page were provided by Kurt Diedrich. If you have any questions you may ask the author (fraktalforschung <email symbol> tele2.de)

