Lecture 3: Uninformed Search

David Pinto

Search Algorithms

- Uninformed Blind search
  - Breadth-first
  - uniform first
  - depth-first
  - Iterative deepening depth-first
  - Bidirectional
  - Branch and Bound

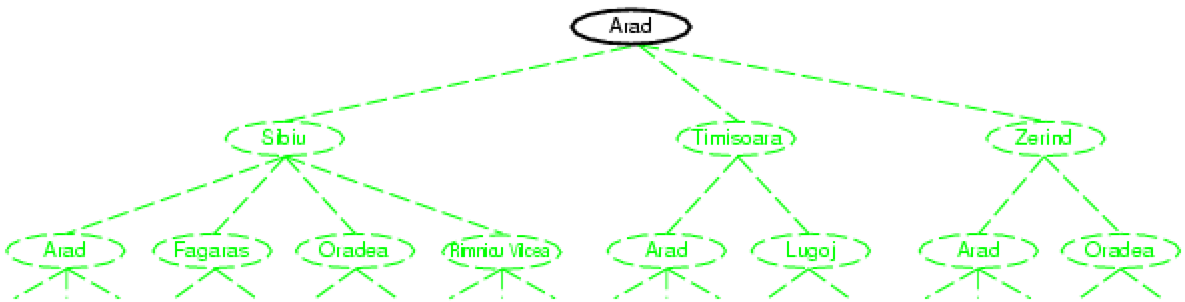
- Informed Heuristic search
  - Greedy search, hill climbing, Heuristics

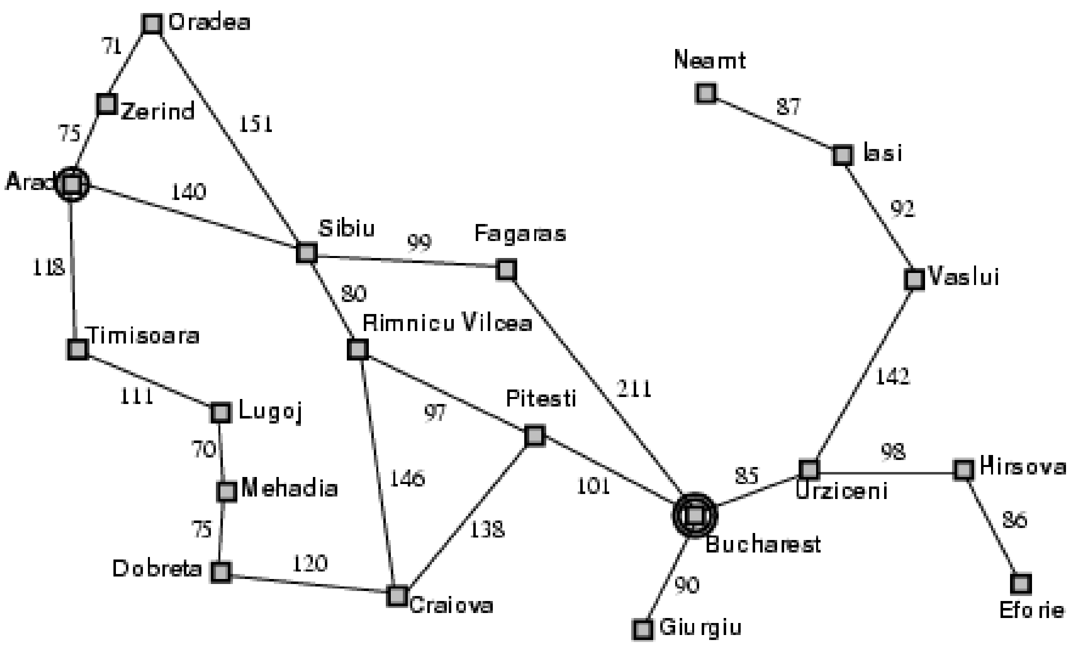
- Important concepts:
  - Completeness
  - Time complexity
  - Space complexity
  - Quality of solution

Tree-based Search

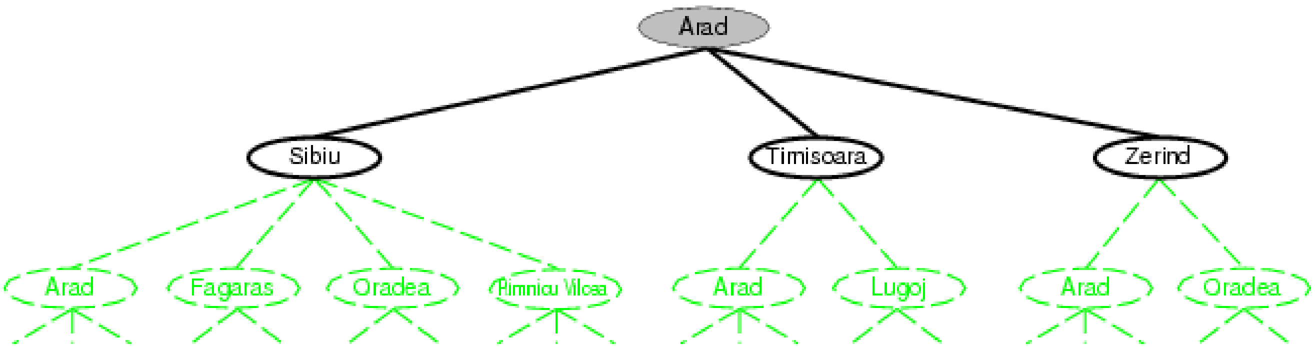
- Basic idea:
  - Exploration of state space by generating successors of already-explored states (a.k.a. expanding states).
  - Every state is evaluated: *is it a goal state*?
- In practice, the solution space can be a graph, not a tree
  - E.g., 8-puzzle
  - More general approach is graph search
  - Tree search can end up repeatedly visiting the same nodes
    - Unless it keeps track of all nodes visited
    - …but this could take vast amounts of memory

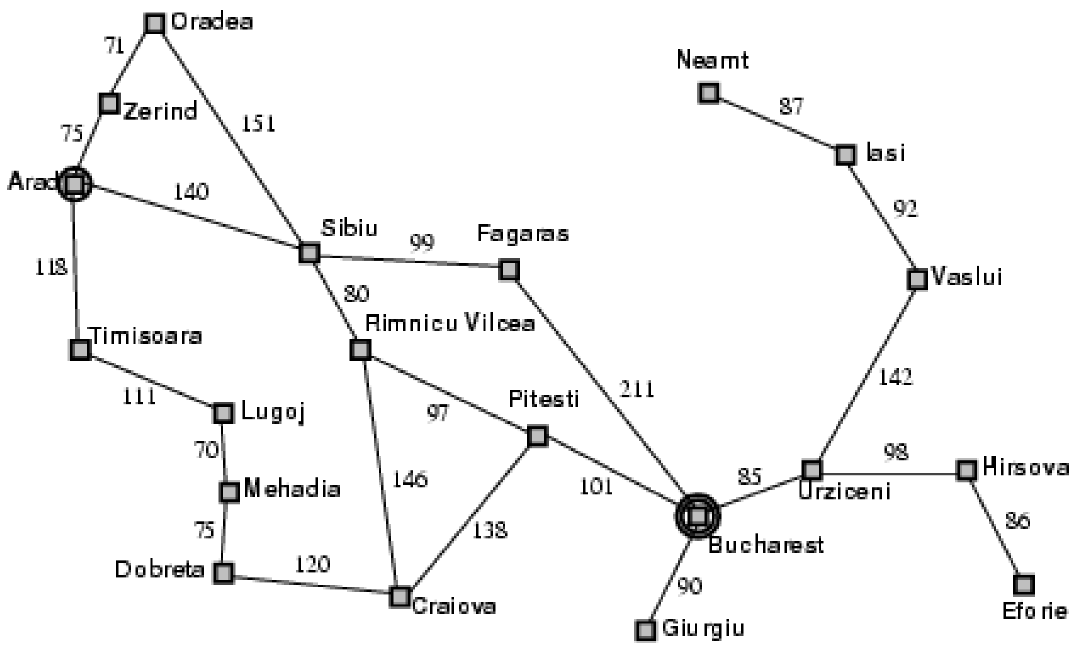
Tree search example



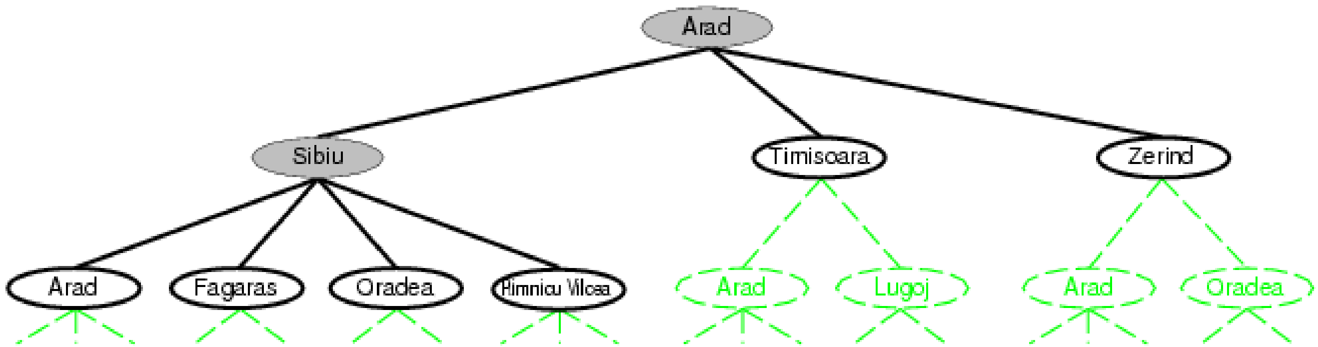


Tree search example



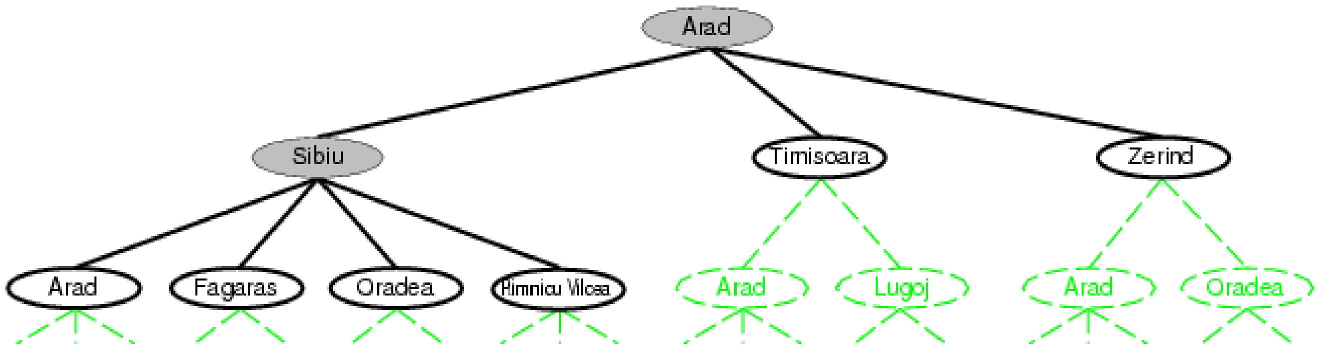


Tree search example



function TREE-SEARCH(*problem, strategy*) **returns** a solution, or failure
  initialize the search tree using the initial state of *problem*
  loop do
    if there are no candidates for expansion **then return** failure
    choose a leaf node for expansion according to *strategy*
    if the node contains a goal state **then return** the corresponding solution
    else expand the node and add the resulting nodes to the search tree

Tree search example



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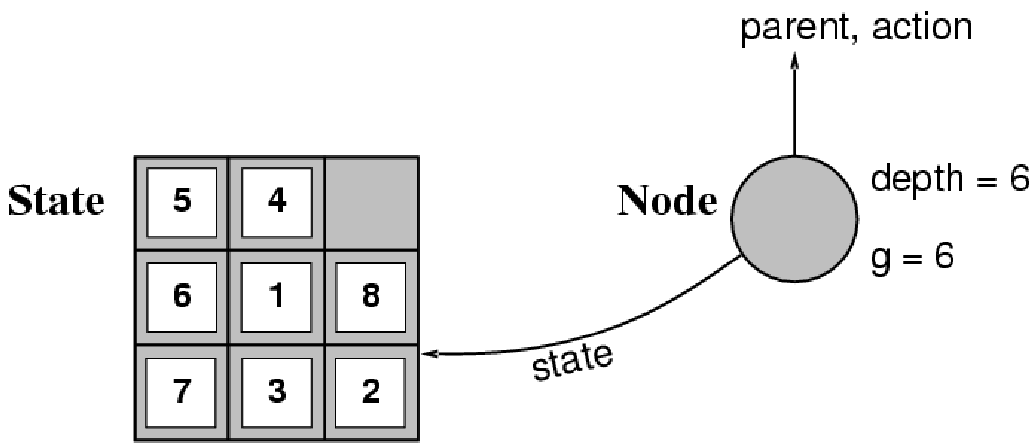
    loop do
        if there are no candidates for expansion **then return** failu
        choose a leaf node for expansion according to *strategy*
        if the node contains a goal state **then return** the corresponding solution
        else expand the node and add the resulting nodes to the search tree

This “strategy” is what differentiates different search algorithms

States versus Nodes

- A state is a (representation of) a physical configuration

- A node is a data structure constituting part of a search tree contains info such as: state, parent node, action, path cost *g(x)*, depth



- The `Expand` function creates new nodes, filling in the various fields and using the `SuccessorFn` of the problem to create the corresponding states.

Search Tree for the 8 puzzle problem

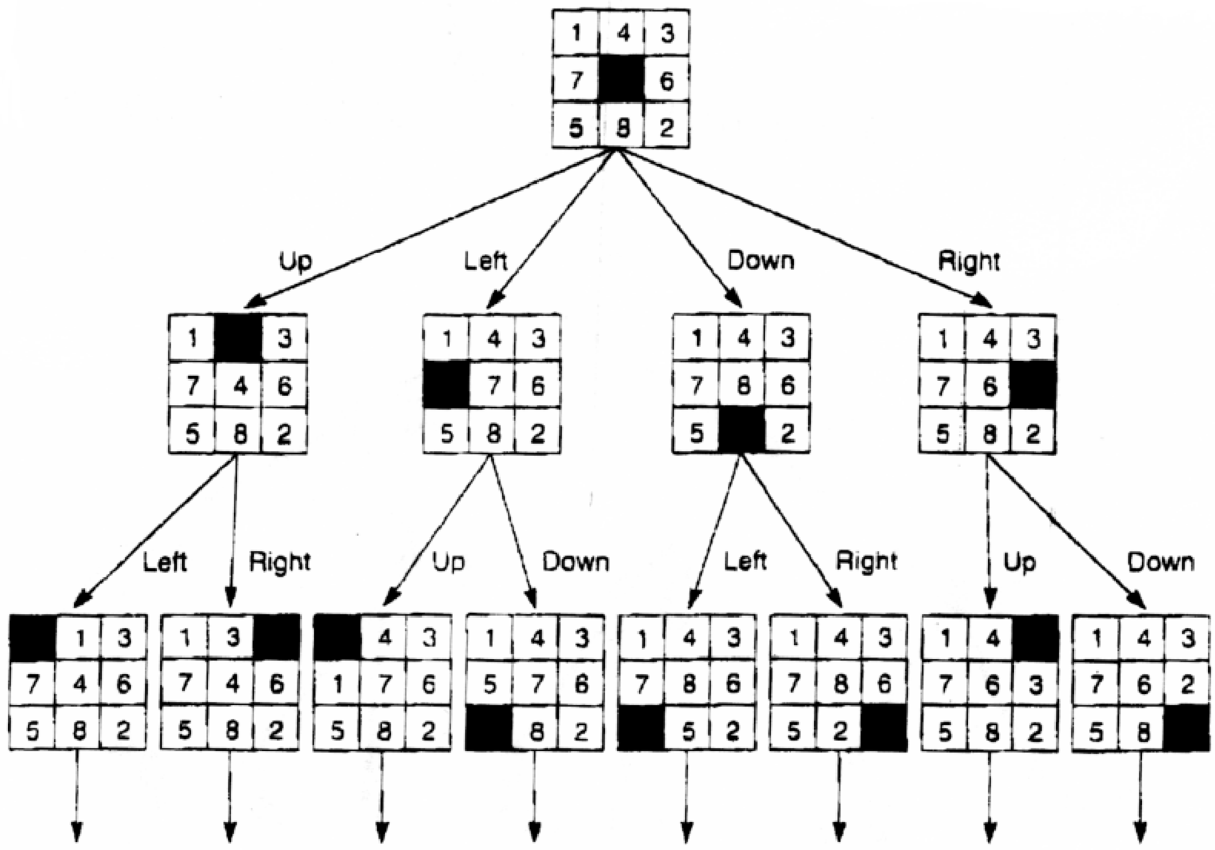


Figure 3.6 State space of the 8-puzzle generated by "move blank" operations.

Search Strategies

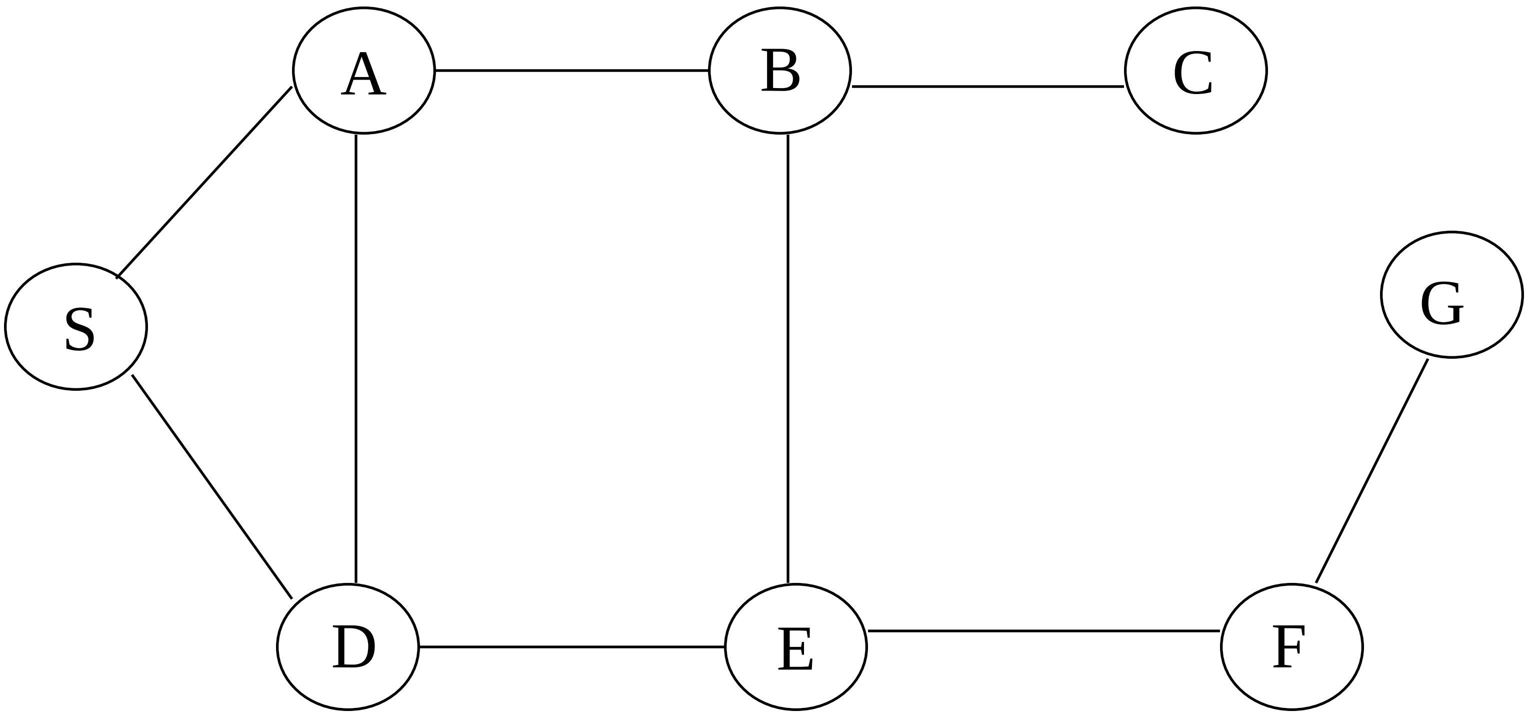
- A search strategy is defined by picking the order of node expansion
- Strategies are evaluated along the following dimensions:
  - completeness: does it always find a solution if one exists?
  - time complexity: number of nodes generated
  - space complexity: maximum number of nodes in memory
  - optimality: does it always find a least-cost solution?
- Time and space complexity are measured in terms of
  - *b:* maximum branching factor of the search tree
  - *d:* depth of the least-cost solution
  - *m*: maximum depth of the state space (may be ∞)

Breadth-First Search (BFS)

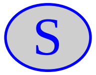
- Expand shallowest unexpanded node
- Fringe: nodes waiting in a queue to be explored, also called OPEN
- Implementation:
  - For BFS, *fringe* is a first-in-first-out (FIFO) queue
  - new successors go at end of the queue
- Repeated states?
  - Simple strategy: do not add parent of a node as a leaf

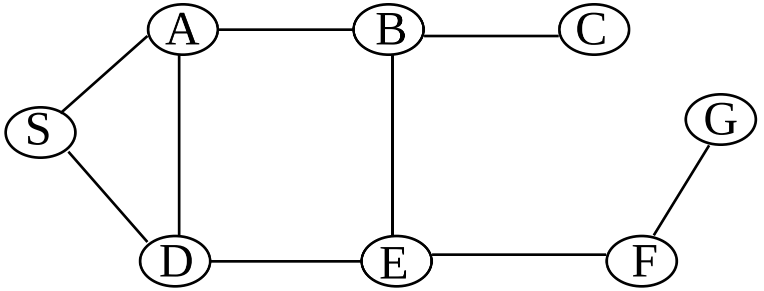
Example: Map Navigation

State Space:
S = start, G = goal, other nodes = intermediate states, links = legal transitions



BFS Search Tree





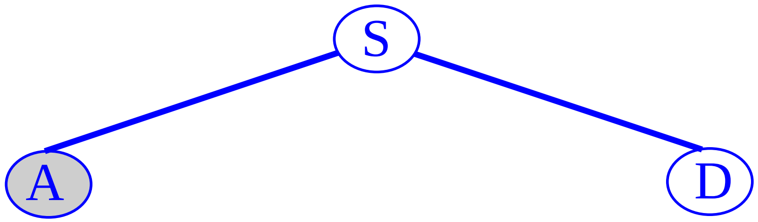
Queue = {S}

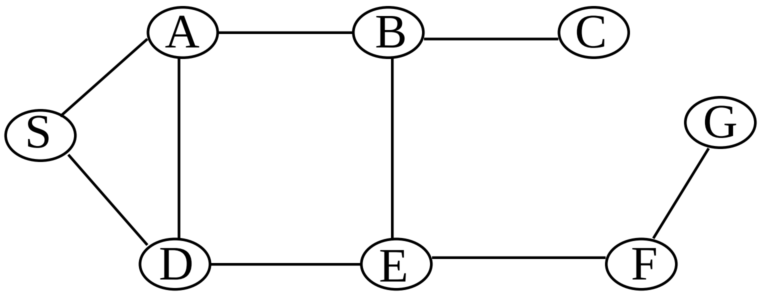
Select S

Goal(S) = true?

If not, Expand(S)

BFS Search Tree





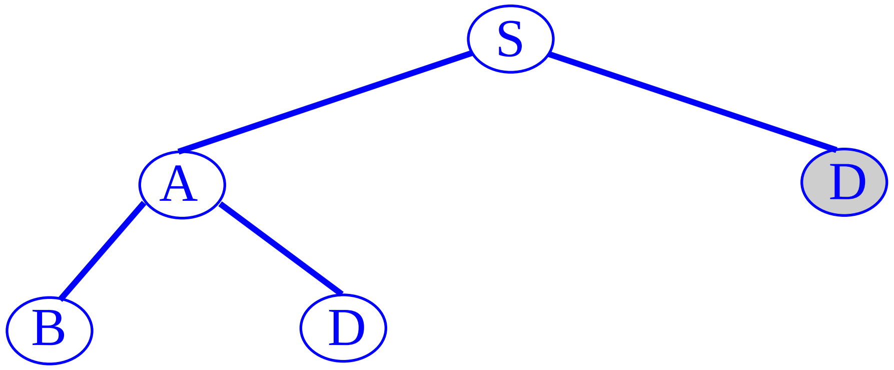
Queue = {A, D}

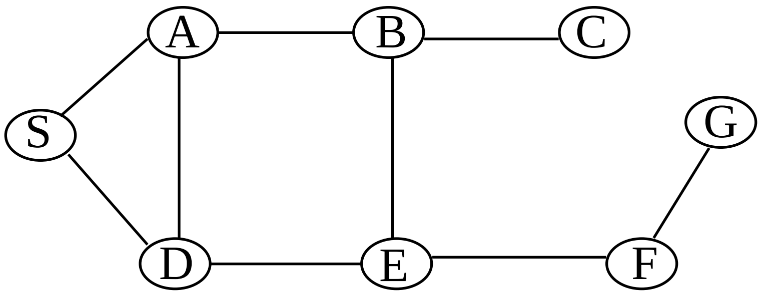
Select A

Goal(A) = true?

If not, Expand(A)

BFS Search Tree





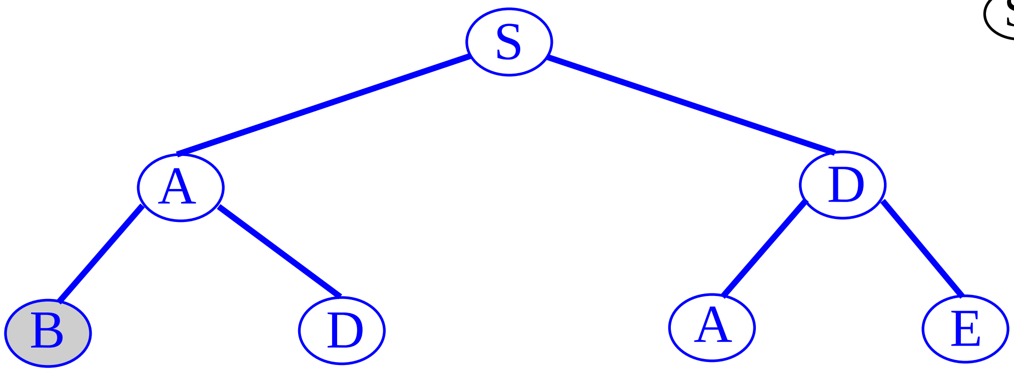
Queue = {D, B, D}

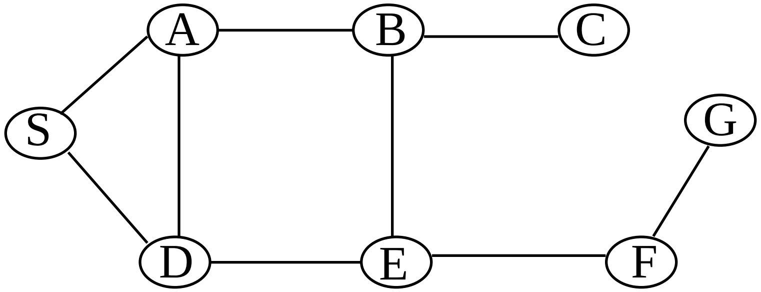
Select D

Goal(D) = true?

If not, expand(D)

BFS Search Tree

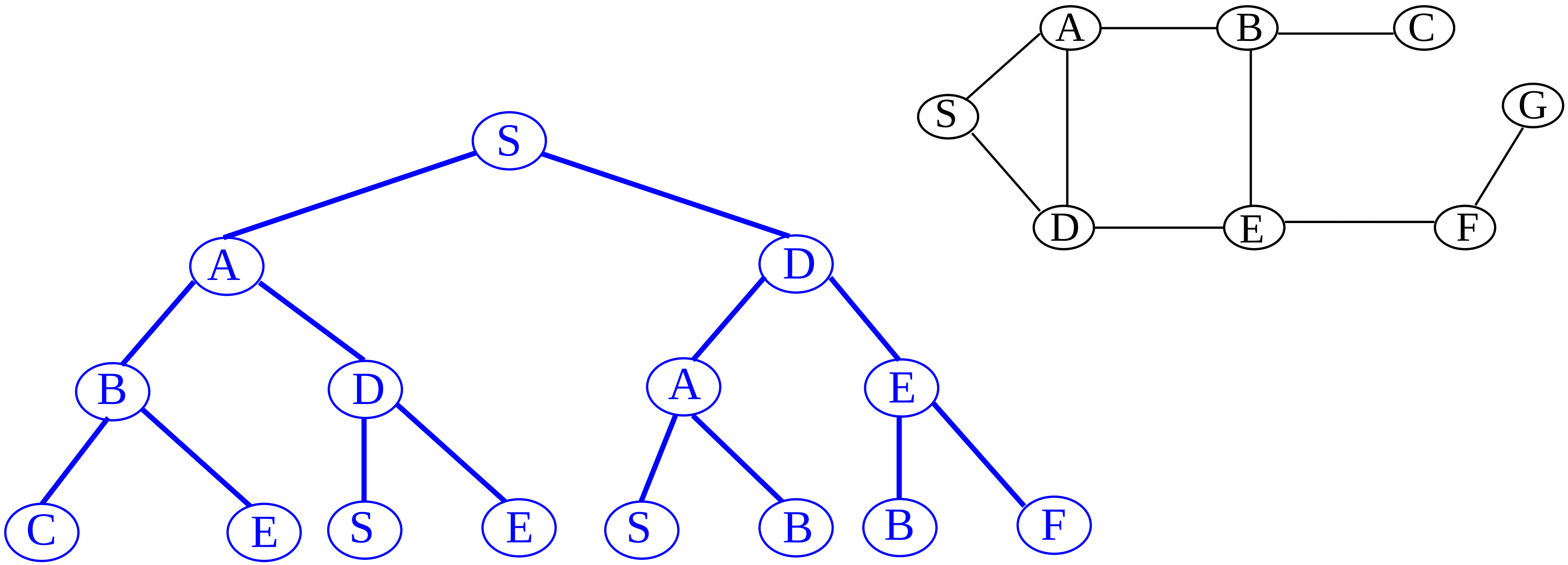




Queue = {B, D, A, E}

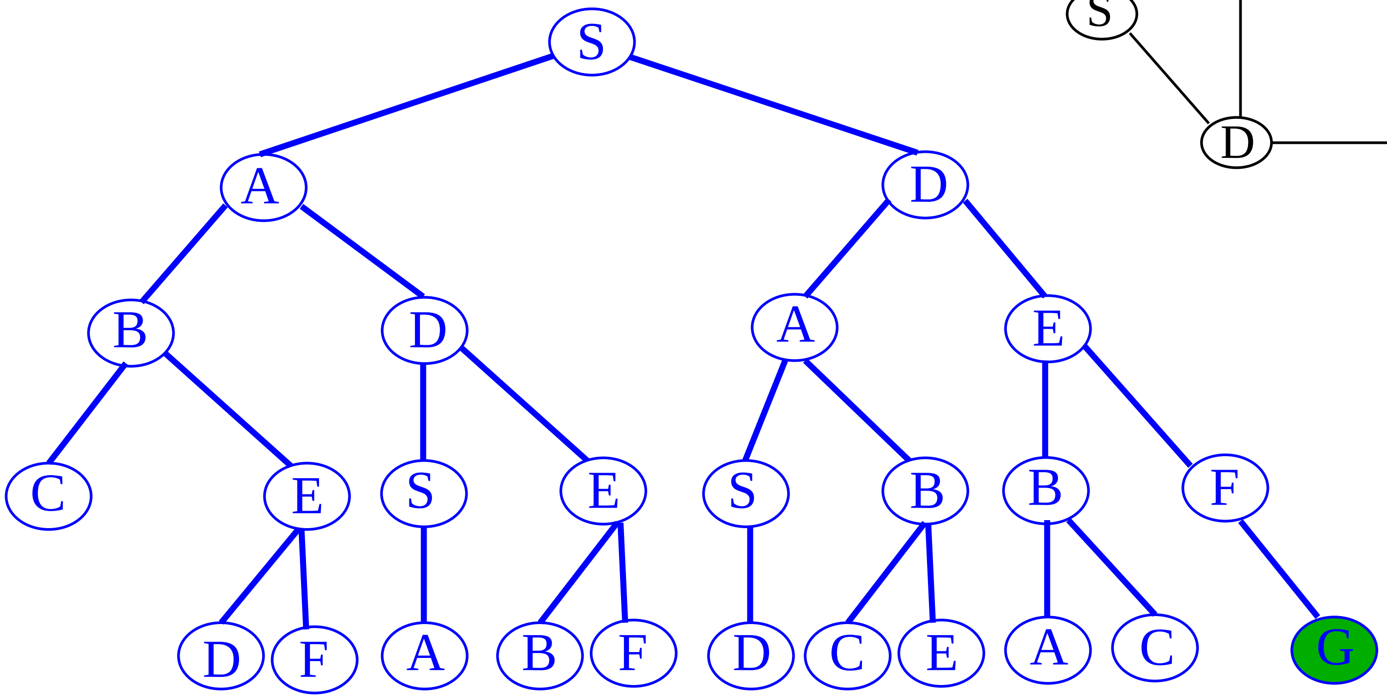
Select B
etc.

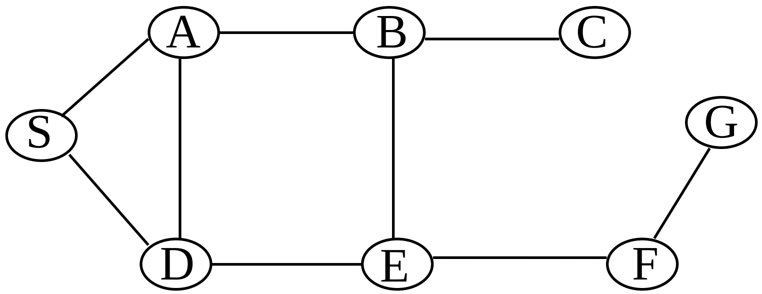
BFS Search Tree



Level 3
Queue = {C, E, S, E, S, B, B, F}

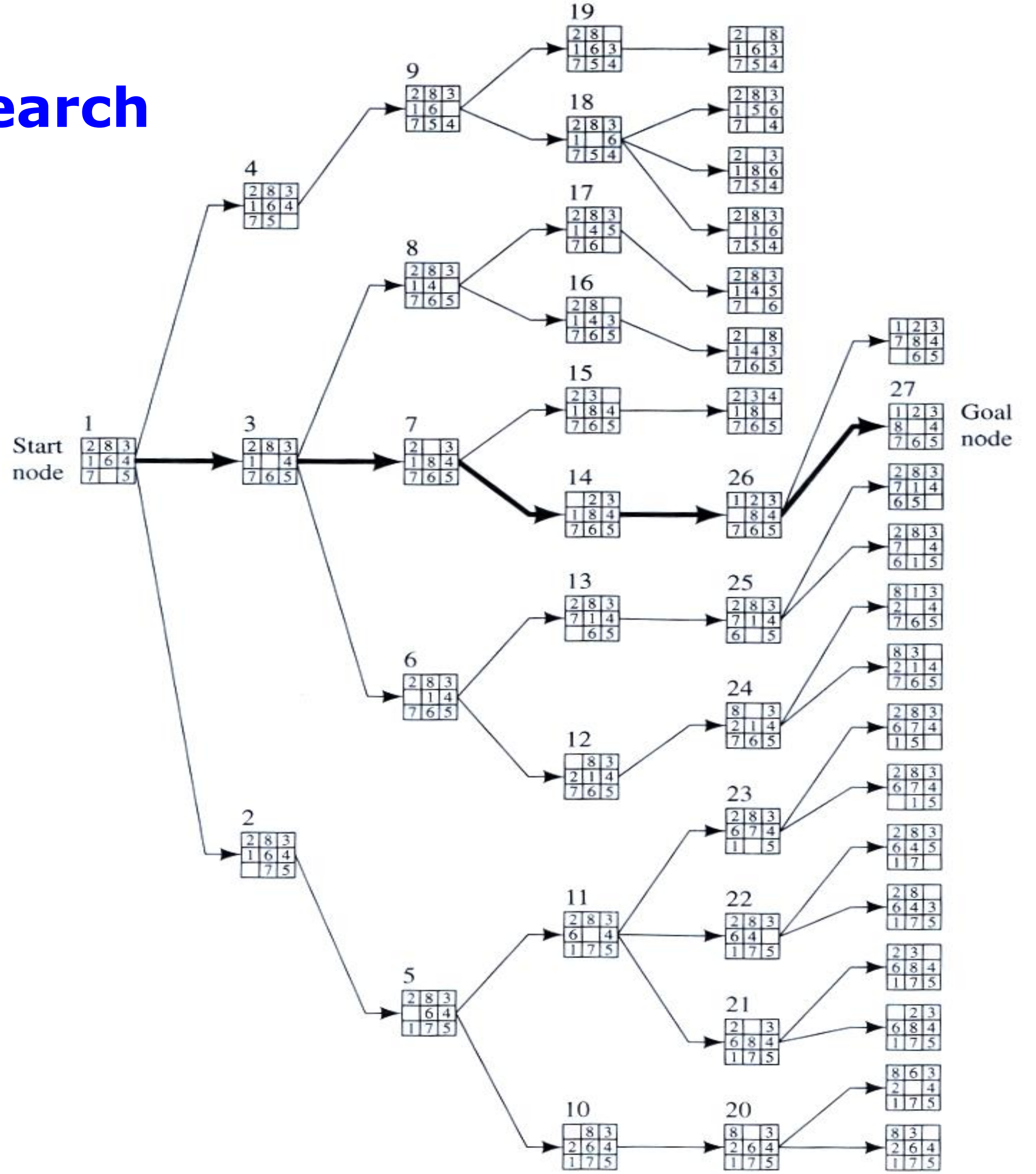
BFS Search Tree





Level 4
Expand queue until G is at front
Select G
Goal(G) = true

Breadth-First Search



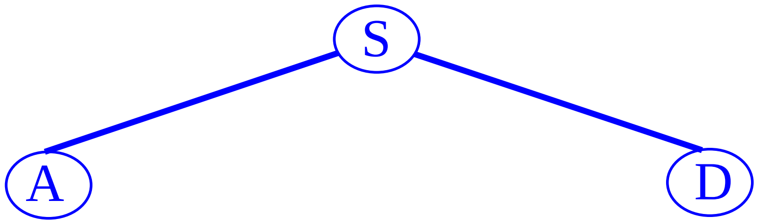
Depth-First Search (BFS)

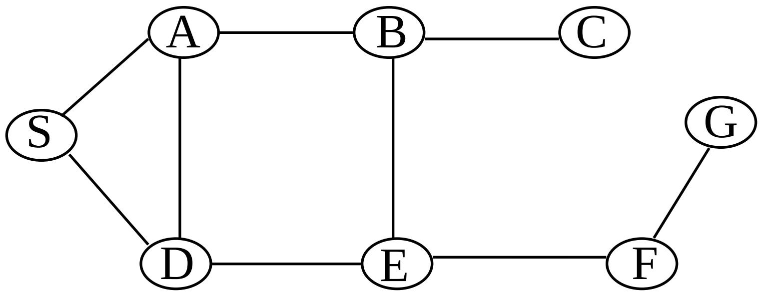
- Expand deepest unexpanded node

- Implementation:
  - For DFS, *fringe* is a first-in-first-out (FIFO) queue
  - new successors go at beginning of the queue

- Repeated nodes?
  - Simple strategy: Do not add a state as a leaf if that state is on the path from the root to the current node

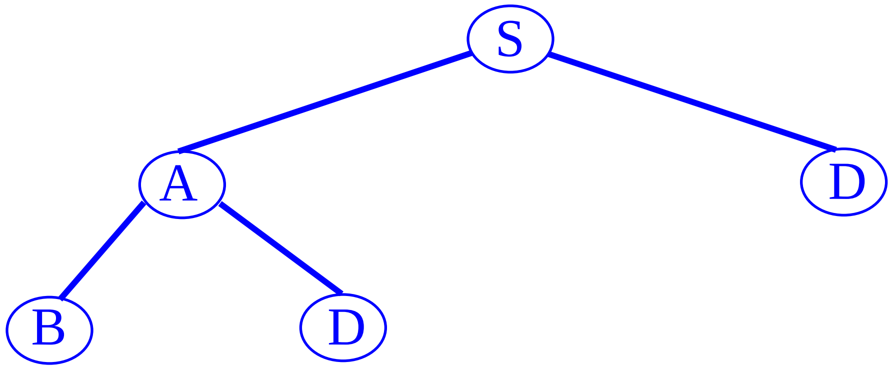
DFS Search Tree

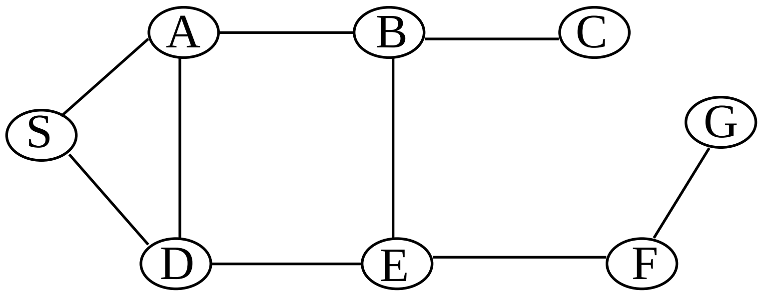




Queue = {A,D}

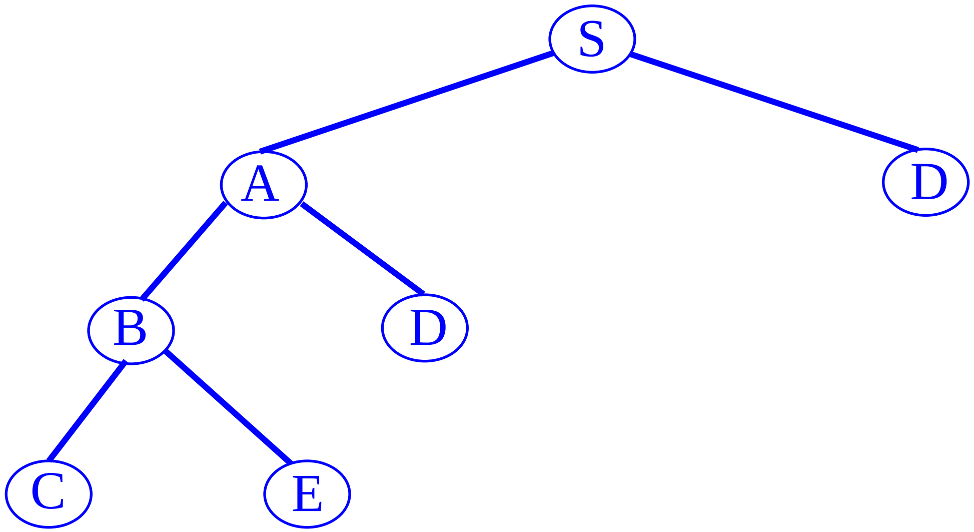
DFS Search Tree

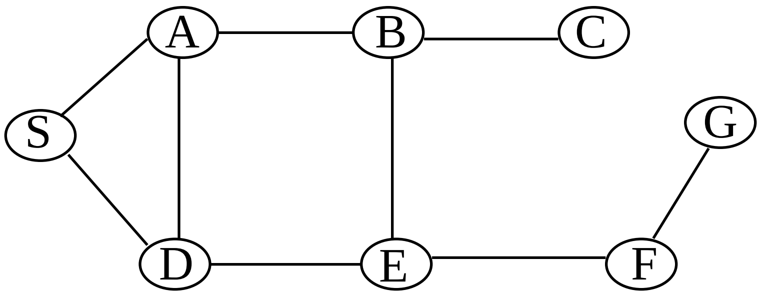




Queue = {B,D,D}

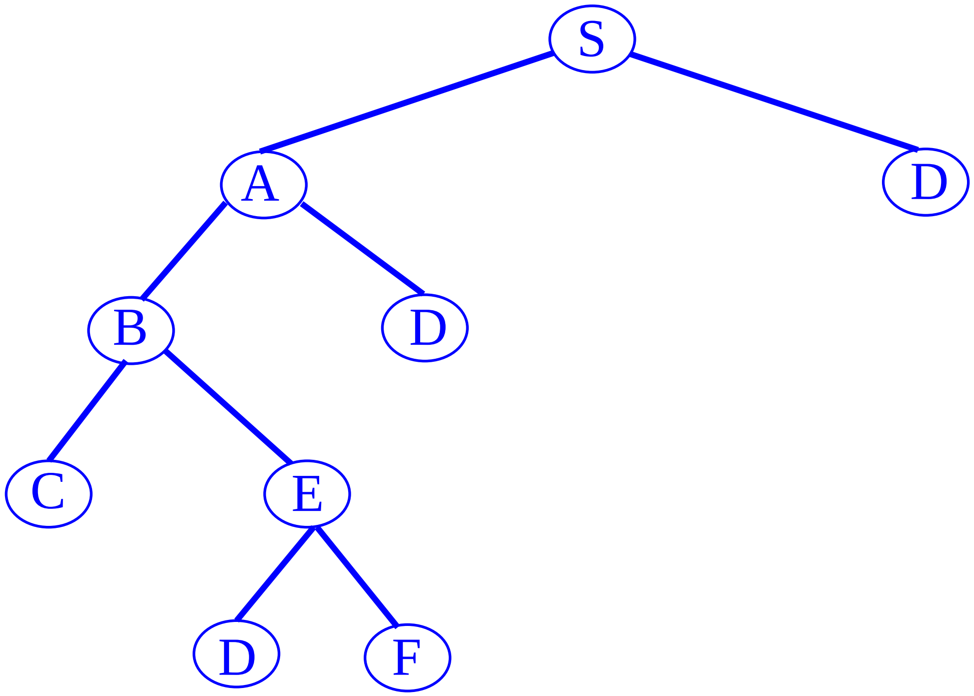
DFS Search Tree

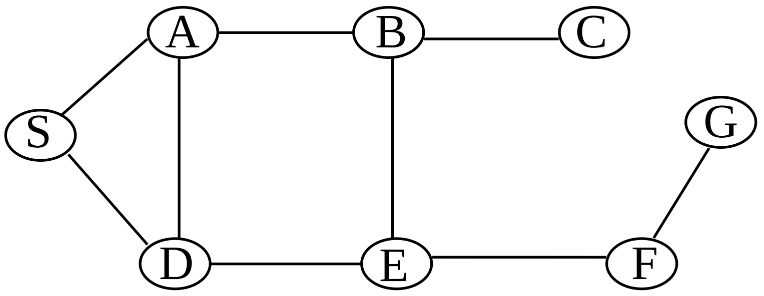




Queue = {C,E,D,D}

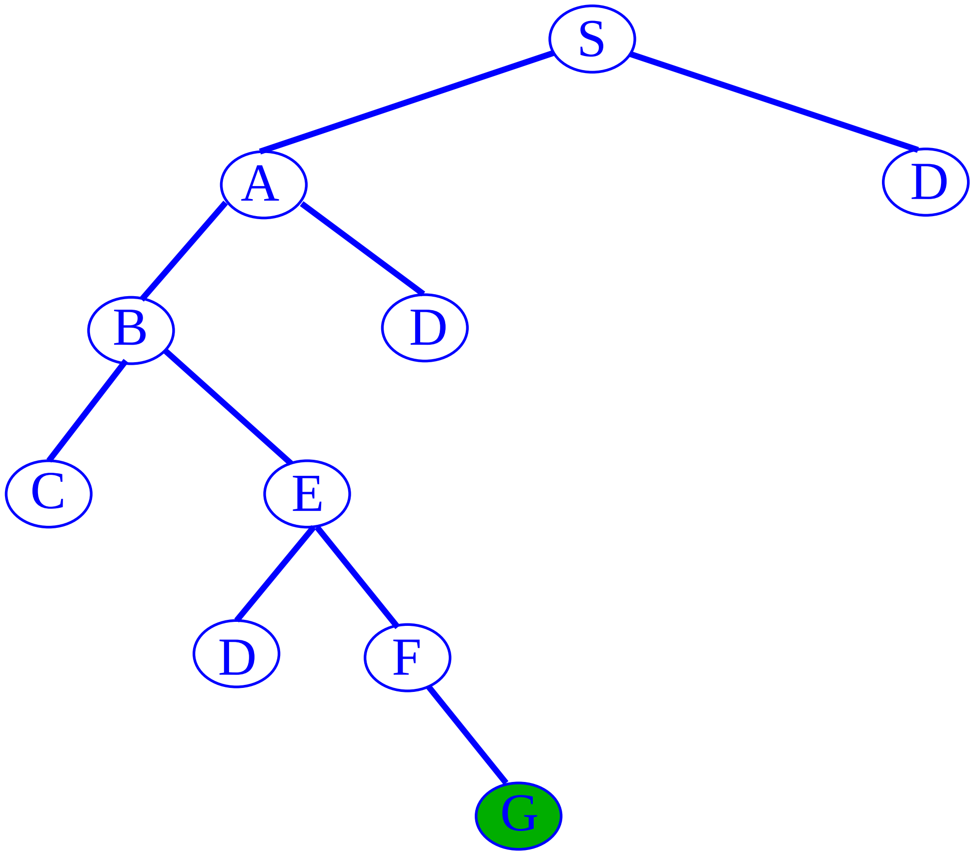
DFS Search Tree

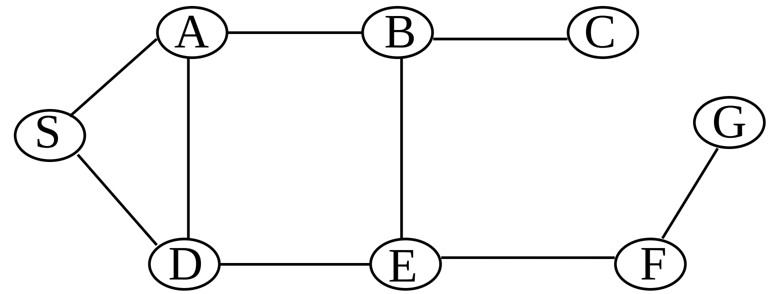




Queue = {D,F,D,D}

DFS Search Tree





Queue = {G,D,D}

Evaluation of Search Algorithms

- Completeness
  - does it always find a solution if one exists?
- Optimality
  - does it always find a least-cost (or min depth) solution?
- Time complexity
  - number of nodes generated (worst case)
- Space complexity
  - number of nodes in memory (worst case)
- Time and space complexity are measured in terms of
  - *b:* maximum branching factor of the search tree
  - *d:* depth of the least-cost solution
  - *m*: maximum depth of the state space (may be ∞)

Breadth-First Search (BFS) Properties

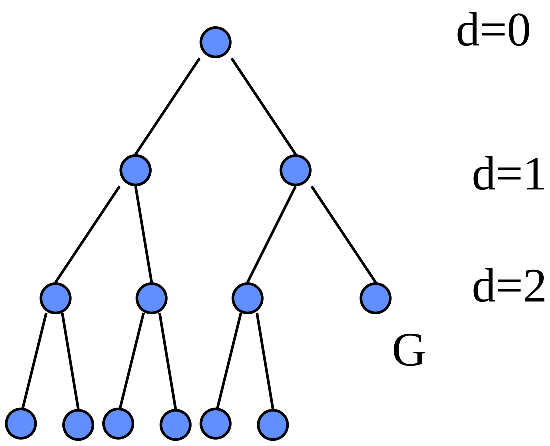
- Complete? Yes
- Optimal? Only if path-cost = non-decreasing function of depth
- Time complexity $O(b^d)$
- Space complexity $O(b^d)$
- Main practical drawback? exponential space complexity

Complexity of Breadth-First Search

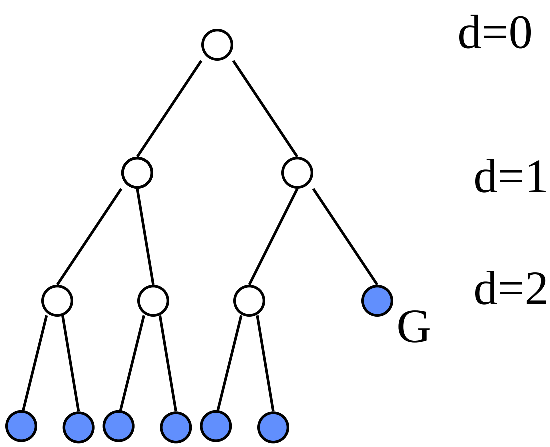
- **Time Complexity**
  - assume (worst case) that there is 1 goal leaf at the RHS at depth d
  - so BFS will generate

    $= b + b^2 + \; ..... + b^d + b^{d+1} - b$

    $= \mathbf{O(b^{d+1})}$



- **Space Complexity**
  - how many nodes can be in the queue (worst-case)?
  - at depth d there are b^{d+1} unexpanded nodes in the Q $= \mathbf{O(b^{d+1})}$



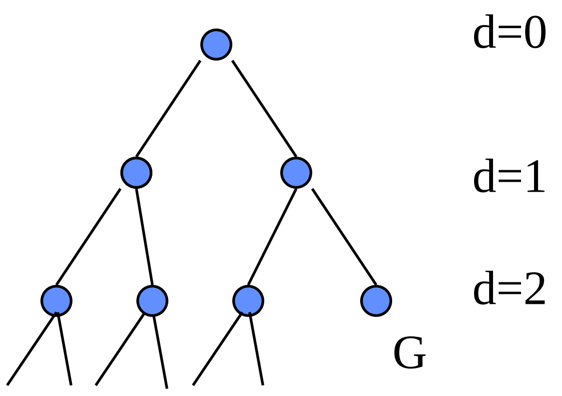
Examples of Time and Memory Requirements for Breadth-First Search

Assuming b=10, 10000 nodes/sec, 1kbyte/node

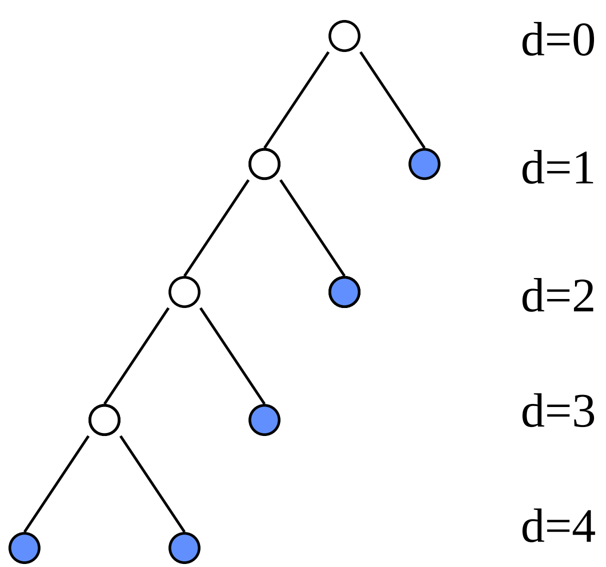
Depth of Solution	Nodes Generated	Time	Memory
2	1100	0.11 seconds	1 MB
4	111,100	11 seconds	106 MB
8	10^9	31 hours	1 TB
12	10^{13}	35 years	10 PB

What is the Complexity of Depth-First Search?

- Time Complexity
  - maximum tree depth = m
  - assume (worst case) that there is 1 goal leaf at the RHS at depth d
  - so DFS will generate **O (b^m)**



- Space Complexity
  - how many nodes can be in the queue (worst-case)?
  - at depth m we have b nodes
  - and b-1 nodes at earlier depths
  - total = b + (m-1)*(b-1) = **O(bm)**



Examples of Time and Memory Requirements for Depth-First Search

Assuming b=10, m = 12, 10000 nodes/sec, 1kbyte/node

Depth of Solution	Nodes Generated	Time	Memory
2	10^{12}	3 years	120kb
4	10^{12}	3 years	120kb
8	10^{12}	3 years	120kb
12	10^{12}	3 years	120kb

Depth-First Search (DFS) Properties

- Complete?
  - Not complete if tree has unbounded depth
- Optimal?
  - No
- Time complexity?
  - Exponential
- Space complexity?
  - Linear

Comparing DFS and BFS

- Time complexity: same, but
  - In the worst-case BFS is always better than DFS
  - Sometime, on the average DFS is better if:
    - many goals, no loops and no infinite paths

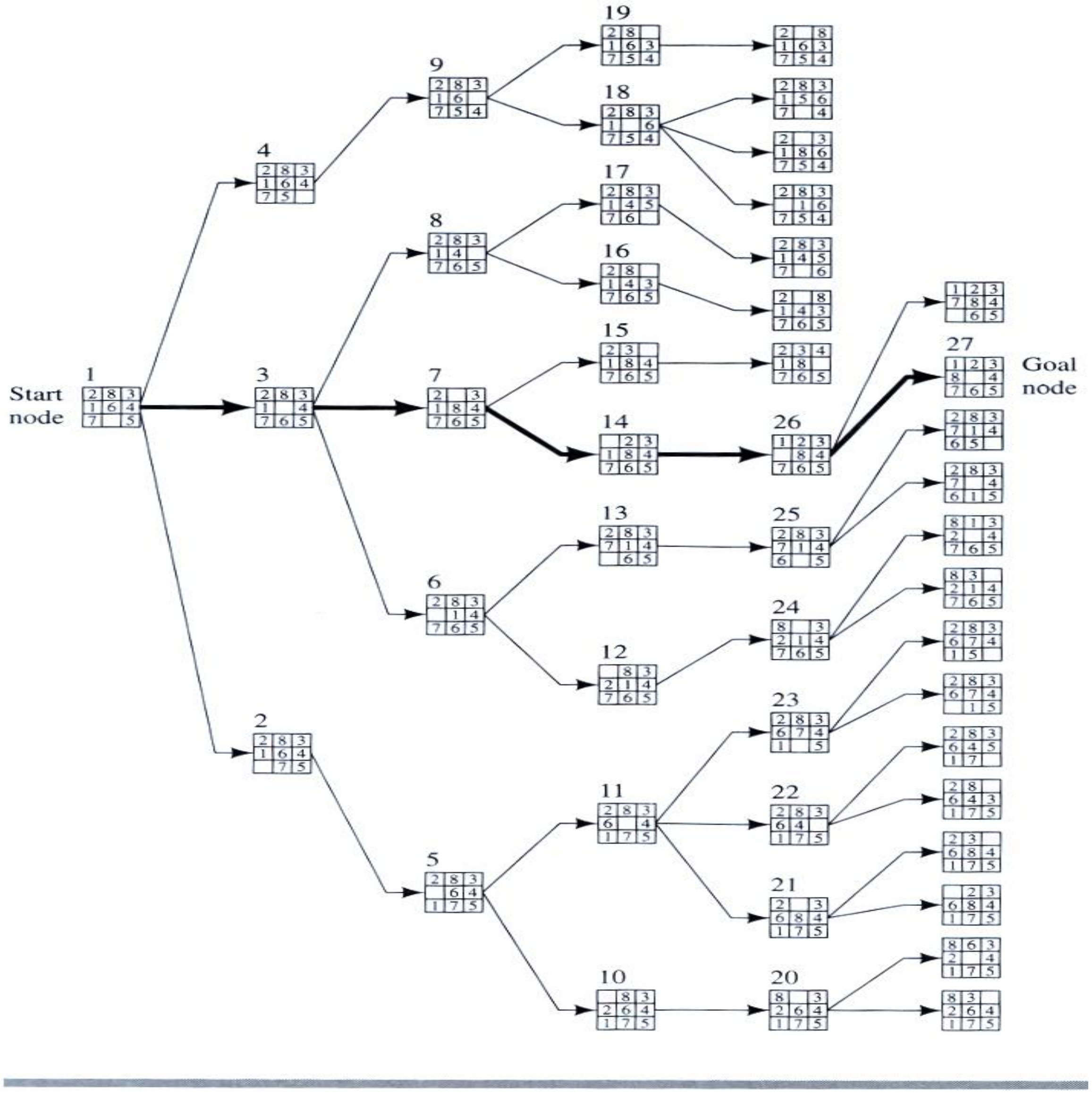
- BFS is much worse memory-wise
    - DFS is linear space
    - BFS may store the whole search space.

- In general
    - BFS is better if goal is not deep, if infinite paths, if many loops, if small search space
    - DFS is better if many goals, not many loops,
    - DFS is much better in terms of memory

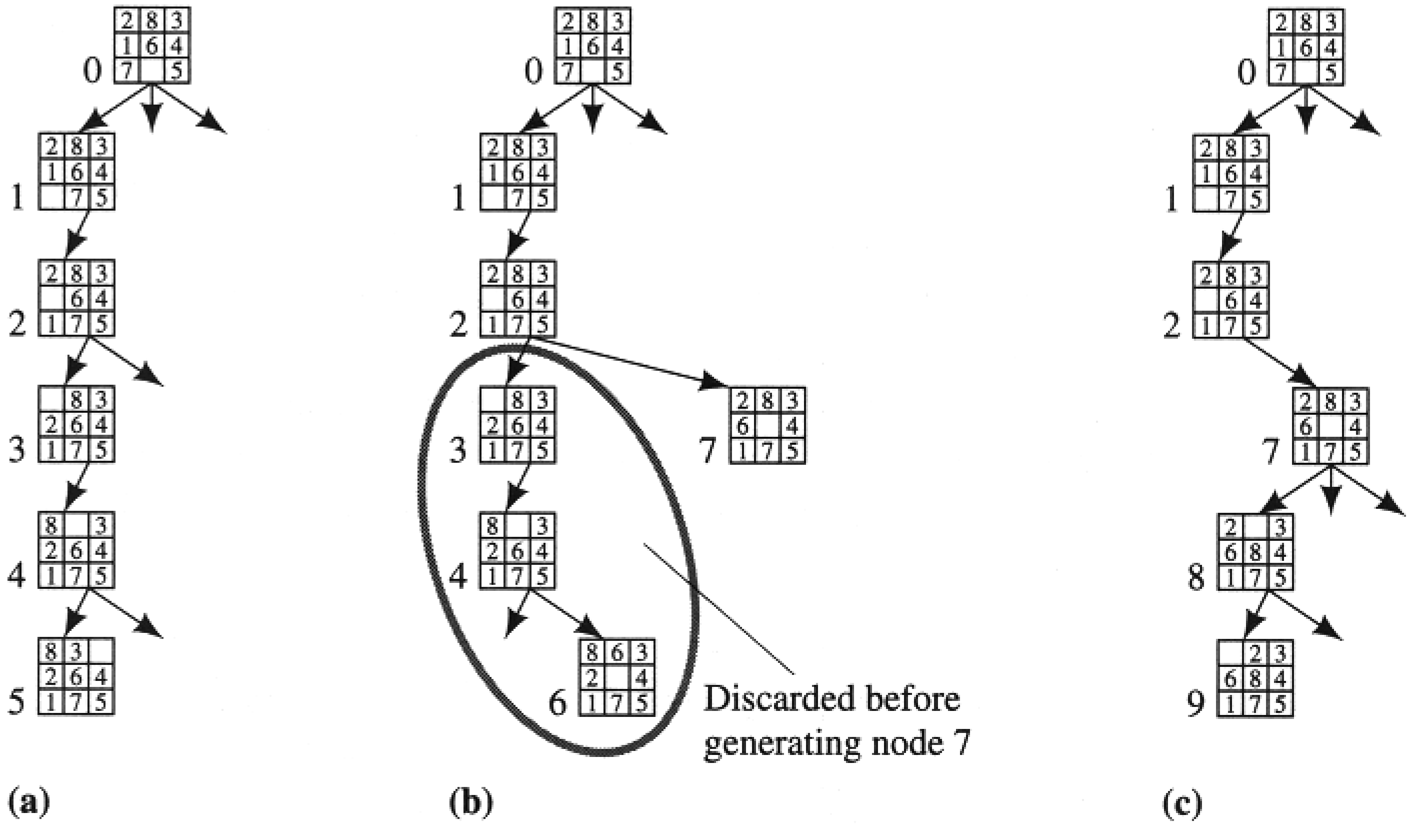
DFS with a depth-limit L

- Standard DFS, but tree is not explored below some depth-limit L
- Solves problem of infinitely deep paths with no solutions
  - But will be incomplete if solution is below depth-limit
- Depth-limit L can be selected based on problem knowledge
  - E.g., diameter of state-space:
    - E.g., max number of steps between 2 cities
  - But typically not known ahead of time in practice

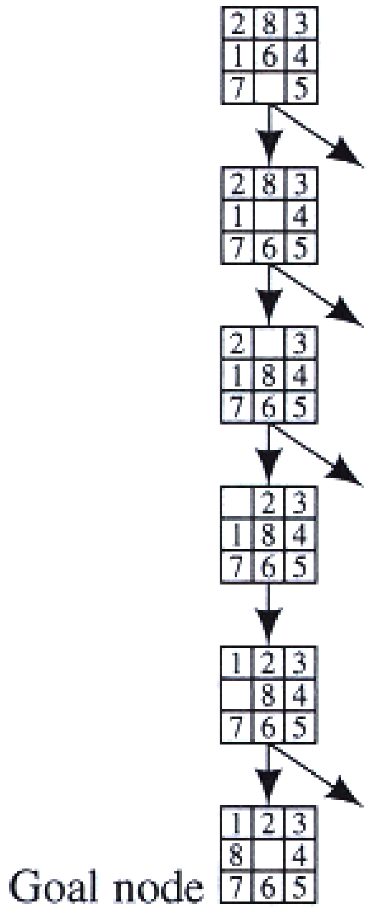
Depth-First Search with a depth-limit, L = 5



Depth-First Search with a depth-limit



Generation of the First Few Nodes in a Depth-First Search



The Graph When the Goal Is Reached in Depth–First Search

Iterative Deepening Search (IDS)

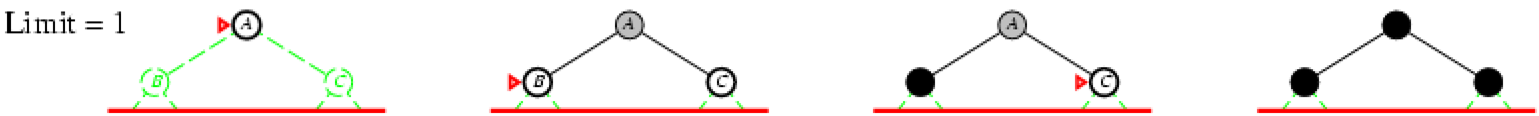
- Run multiple DFS searches with increasing depth-limits

Iterative deepening search

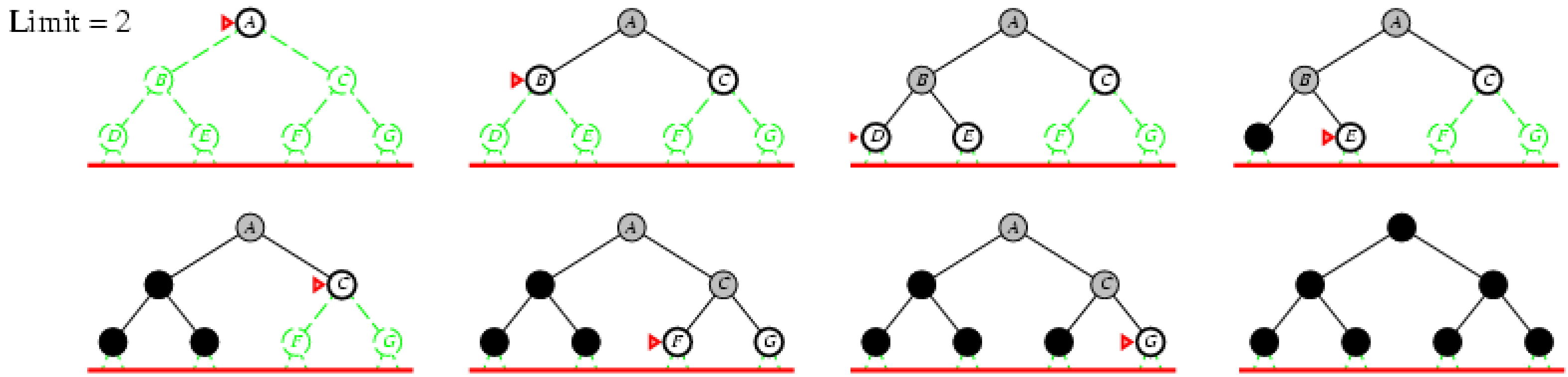
- L = 1
- While no solution, do
  - DFS from initial state S_0 with cutoff L
  - If found goal,
  - stop and return solution,
  - else, increment depth limit L

Iterative deepening search *L*=0

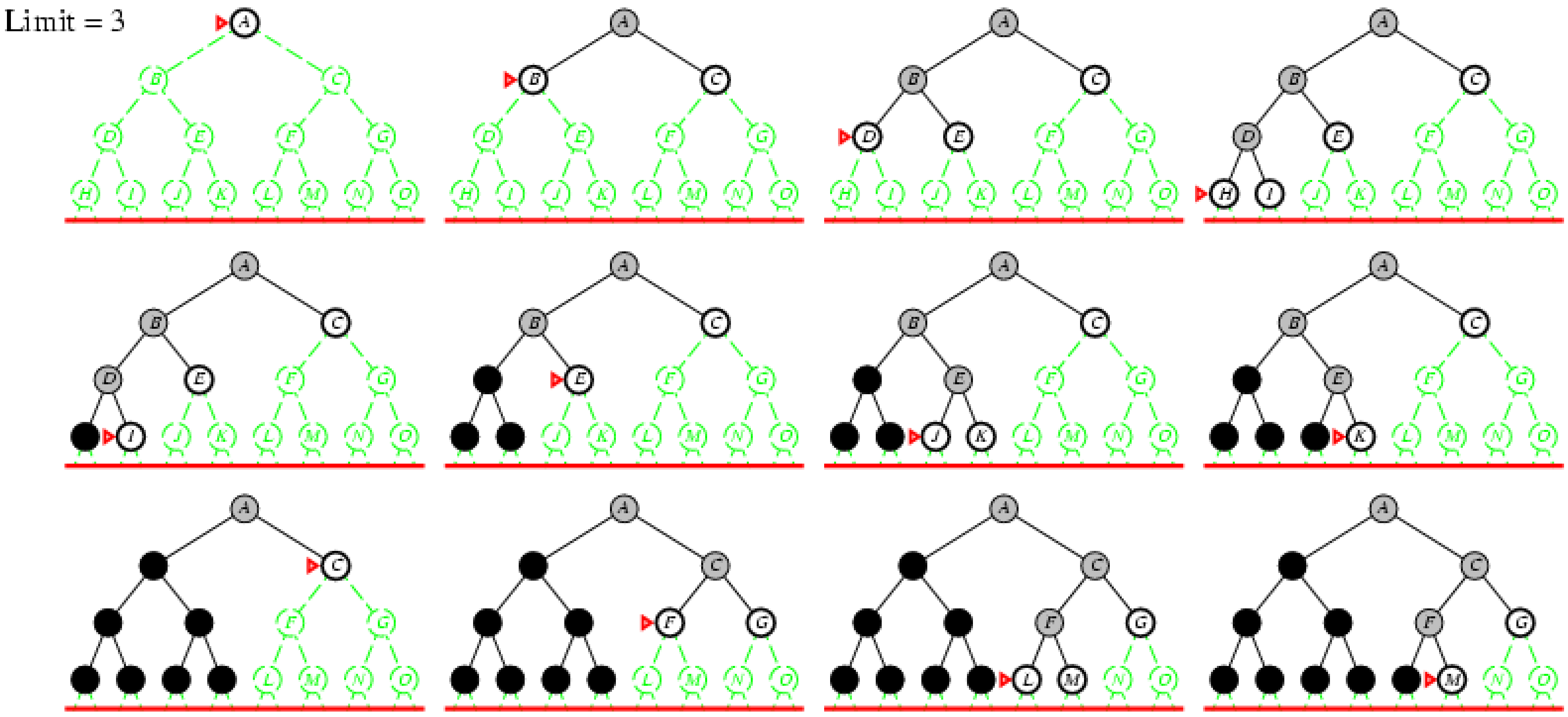
Iterative deepening search *L*=1



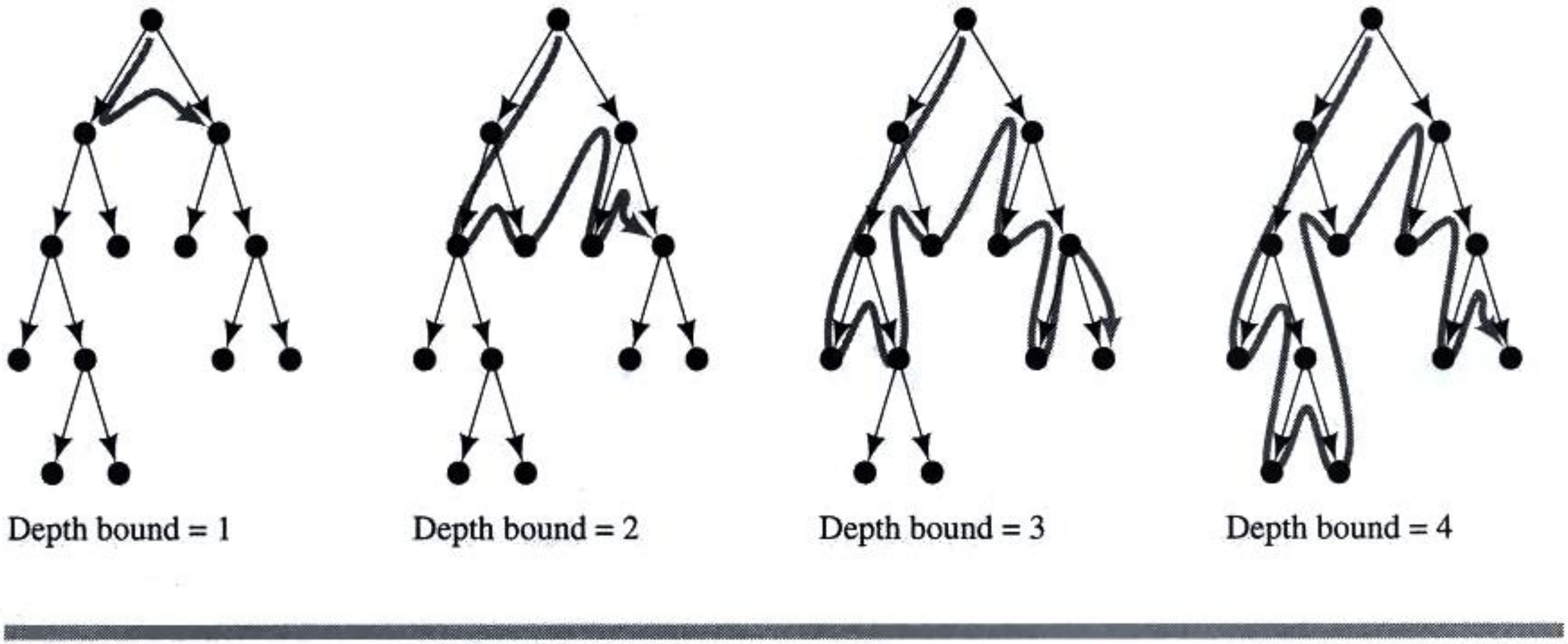
Iterative deepening search $L=2$



Iterative Deepening Search *L*=3



Iterative deepening search



Stages in Iterative-Deepening Search

Properties of Iterative Deepening Search

- Space complexity = O(bd)
    - (since its like depth first search run different times, with maximum depth limit d)

- Time Complexity
    - $b + (b+b^2) + ...(b+....b^d) = O(b^d)$
      (i.e., asymptotically the same as BFS or DFS to limited depth d in the worst case)

- Complete?
    - Yes

- Optimal
    - Only if path cost is a non-decreasing function of depth

- IDS combines the small memory footprint of DFS, and has the completeness guarantee of BFS

IDS in Practice

- Isn't IDS wasteful?
  - Repeated searches on different iterations

  - Compare IDS and BFS:
    - E.g., $b = 10$ and $d = 5$

    - $N(IDS) \sim db + (d-1)b^2 + \ldots\ldots b^d = 123{,}450$

    - $N(BFS) \sim b + b^2 + \ldots\ldots b^d = 111{,}110$

    - Difference is only about 10%
      - Most of the time is spent at depth d, which is the same amount of time in both algorithms

- In practice, IDS is the preferred uniform search method with a large search space and unknown solution depth

Bidirectional Search

- Idea
  - simultaneously search forward from S and backwards from G
  - stop when both "meet in the middle"
  - need to keep track of the intersection of 2 open sets of nodes
- What does searching backwards from G mean
  - need a way to specify the predecessors of G
    - this can be difficult,
    - e.g., predecessors of checkmate in chess?
  - what if there are multiple goal states?
  - what if there is only a goal test, no explicit list?
- Complexity
  - time complexity at best is: $O(2\ b^{(d/2)}) = O(b^{(d/2)})$
  - memory complexity is the same

Bi-Directional Search

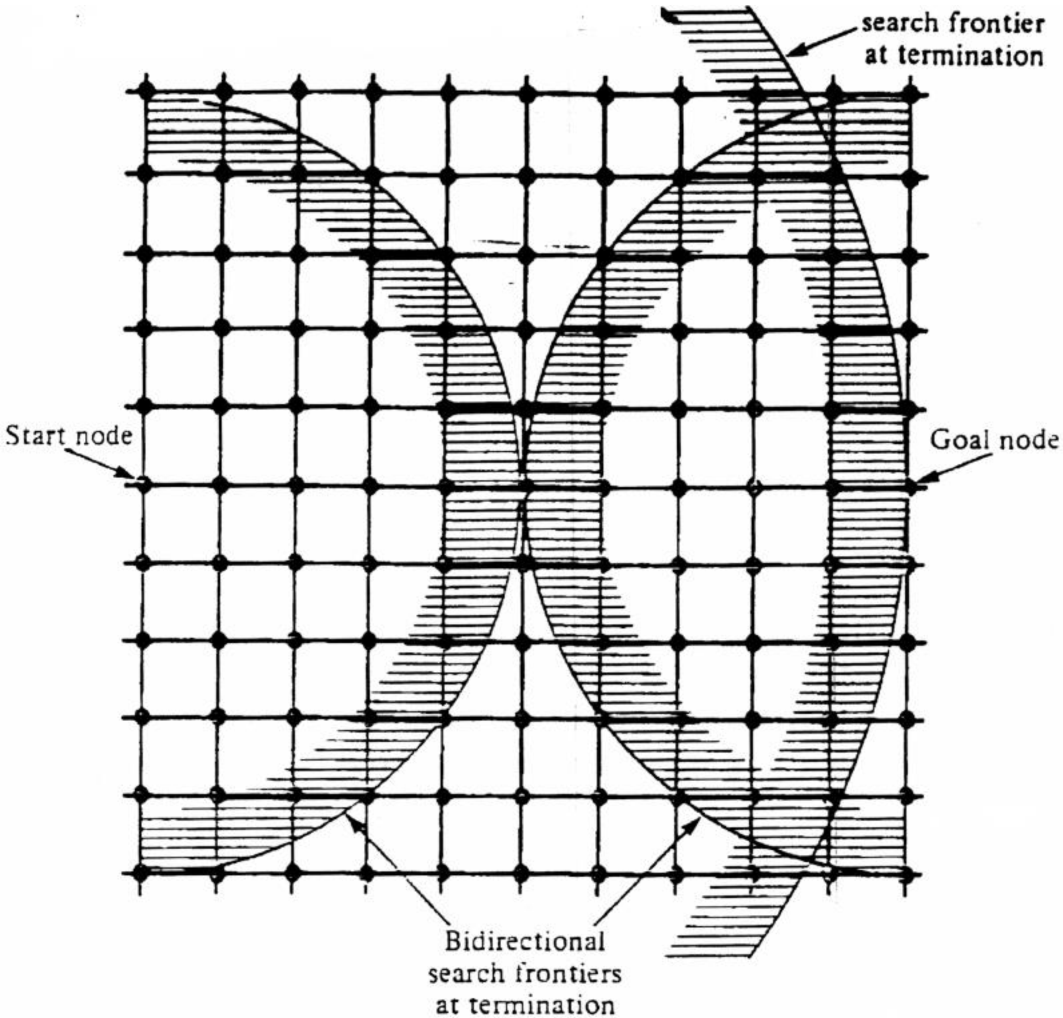


Fig. 2.10 Bidirectional and unidirectional breadth-first searches.

Uniform Cost Search

- Optimality: path found = lowest cost
  - Algorithms so far are only optimal under restricted circumstances
- Let g(n) = cost from start state S to node n
- Uniform Cost Search:
  - Always expand the node on the fringe with minimum cost g(n)
  - Note that if costs are equal (or almost equal) will behave similarly to BFS

Uniform Cost Search

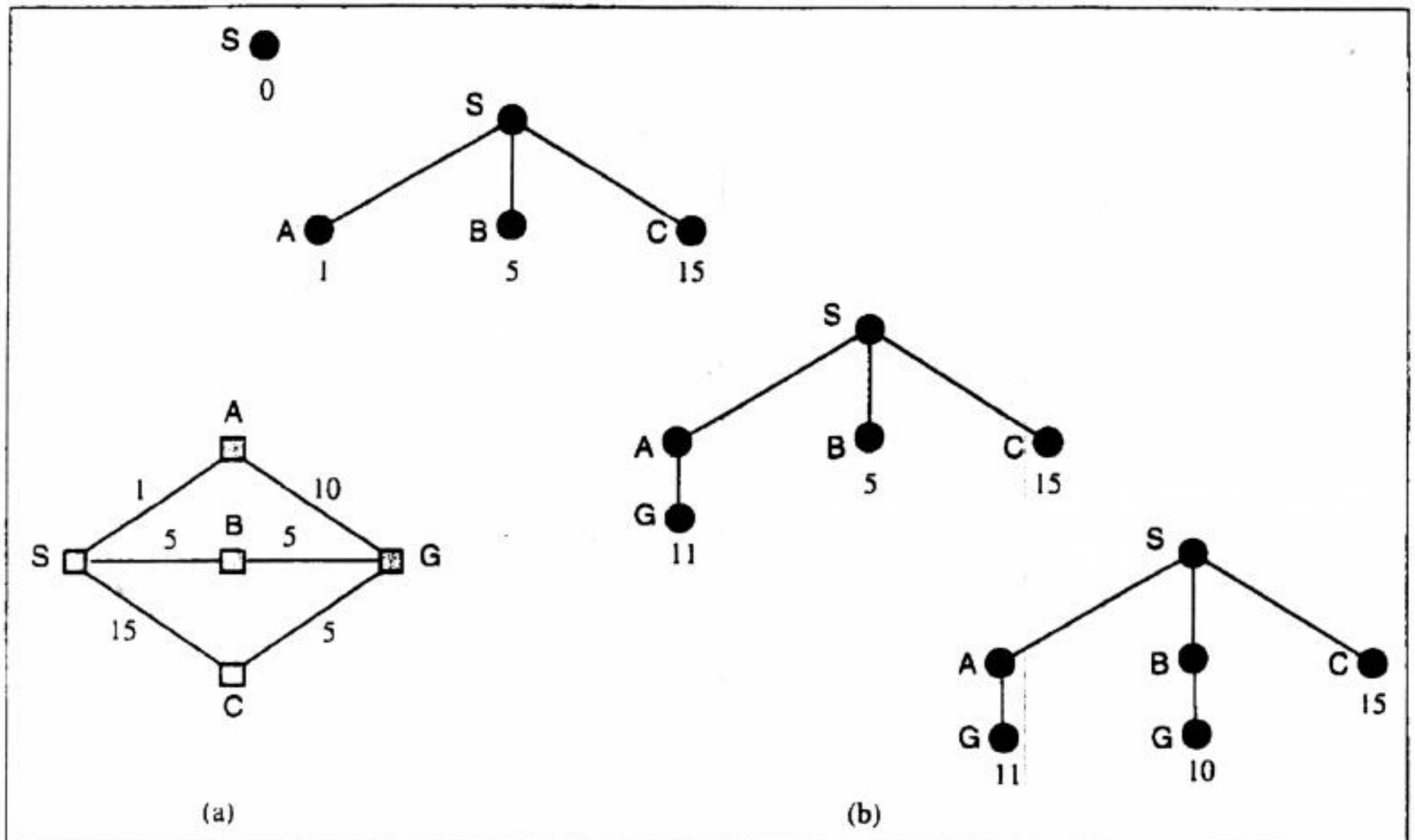


Figure 3.13 A route-finding problem. (a) The state space, showing the cost for each operator. (b) Progression of the search. Each node is labelled with $g(n)$. At the next step, the goal node with $g = 10$ will be selected.

Optimality of Uniform Cost Search?

- Assume that every step costs at least $\varepsilon > 0$

- Proof of Completeness:

  Given that every step will cost more than 0, and assuming a finite branching factor, there is a finite number of expansions required before the total path cost is equal to the path cost of the goal state. Hence, we will reach it in a finite number of steps.

- Proof of Optimality given Completeness:
  - Assume UCS is not optimal.
  - Then there must be a goal state with path cost smaller than the goal state which was found (invoking completeness)
  - However, this is impossible because UCS would have expanded that node first by definition.
  - Contradiction.

Complexity of Uniform Cost

- Let C* be the cost of the optimal solution

- Assume that every step costs at least $\varepsilon > 0$

- Worst-case time and space complexity is:

$$O(\ b^{\,[1 + \mathrm{floor}(C^*/\varepsilon)]}\ )$$

Why?

floor(C*/ε) ~ depth of solution if all costs are approximately equal

Comparison of Uninformed Search Algorithms

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening
Complete?	Yes	Yes	No	No	Yes
Time	$O(b^{d+1})$	$O(b^{\lceil C^*/\epsilon \rceil})$	$O(b^m)$	$O(b^l)$	$O(b^d)$
Space	$O(b^{d+1})$	$O(b^{\lceil C^*/\epsilon \rceil})$	$O(bm)$	$O(bl)$	$O(bd)$
Optimal?	Yes	Yes	No	No	Yes

Average case complexity of these algorithms?

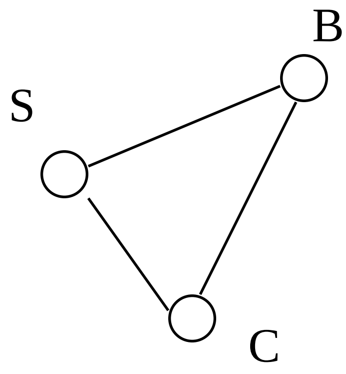
- How would we do an average case analysis of these algorithms?

- E.g., single goal in a tree of maximum depth m
  - Solution randomly located at depth d?
  - Solution randomly located in the search tree?
  - Solution randomly located in state-space?

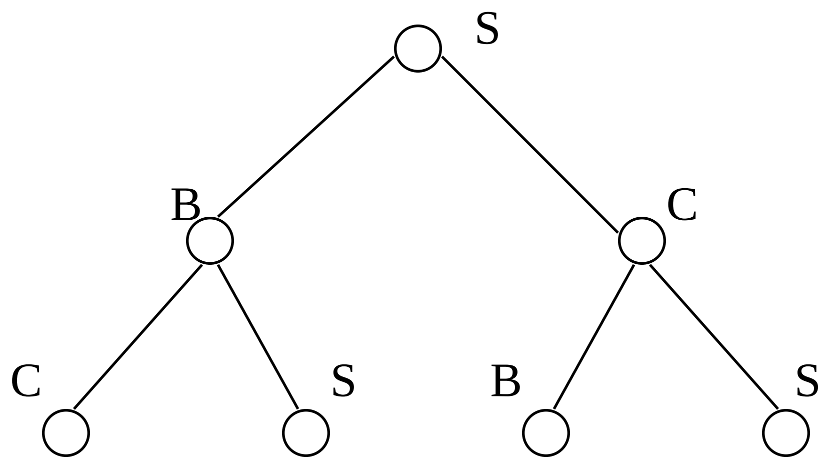
  - What about multiple solutions?

[left as an exercise for the student]

Avoiding Repeated States



State Space



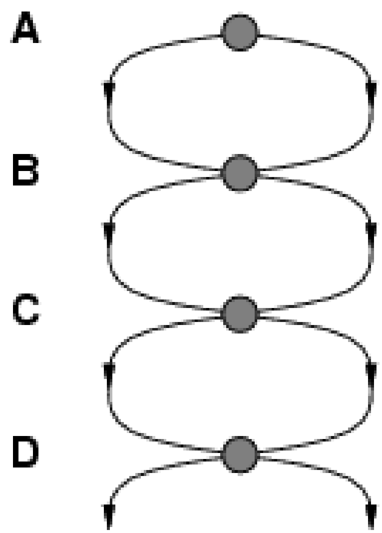
Example of a Search Tree

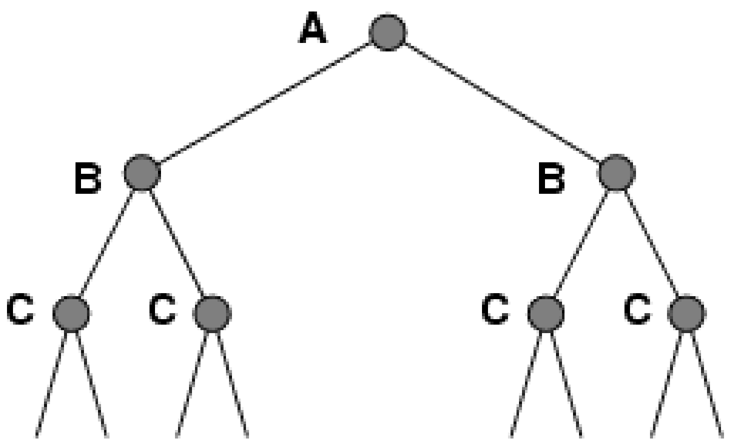
- Possible solution
  - do not add nodes that are on the path from the root
    - Avoids paths containing cycles (loops)
  - easy to check in DFS

- Avoids infinite-depth trees (for finite-state problems) but does not avoid visiting the same states again in other branches

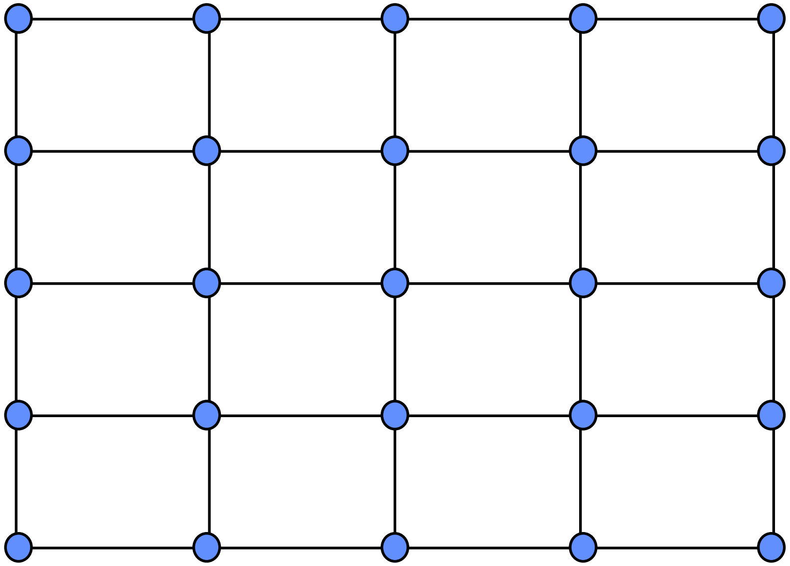
Repeated States

- Failure to detect repeated states can turn a linear problem into an exponential one!





Grid Search: many paths to the same states



- Grid structure to state space
  - Each state has b = 4 successors
    - So full search tree is size 4^d
  - But there are only $2d^2$ distinct states within d steps of any state

  - E.g., d = 20: 10^{12} nodes in search tree, but only 800 distinct states

Graph Search v. Tree Search

- Record every state visited and only generate states that are not on this list

- Modify Tree-Search algorithm
  - Add a data structure called <u>closed-list</u>
    - Stores every previously expanded node
  - (fringe of unexpanded nodes is called the <u>open-list</u>)

  - If current node is on the closed-list, it is discarded, not expanded

  - Can have exponential memory requirements

  - However, on problems with many repeated states (but small state-space), graph-search can be much more efficient than tree-search.

Summary

- A review of search
  - a search space consists of states and operators: it is a graph
  - a search tree represents a particular exploration of search space
- There are various strategies for “uninformed search”
  - breadth-first
  - depth-first
  - iterative deepening
  - bidirectional search
  - Uniform cost search
- Various trade-offs among these algorithms
  - “best” algorithm will depend on the nature of the search problem
- Methods for detecting repeated states
- Next up – heuristic search methods